Formation of sunspots

[turbulence] [stratification] [magnetic field]

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Flux tubes

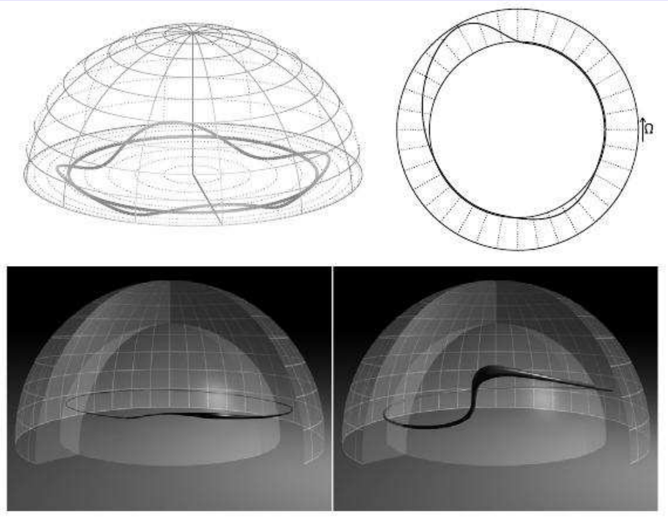


Figure 1 : Caligari et al. (1995)

Flux tubes: problems.

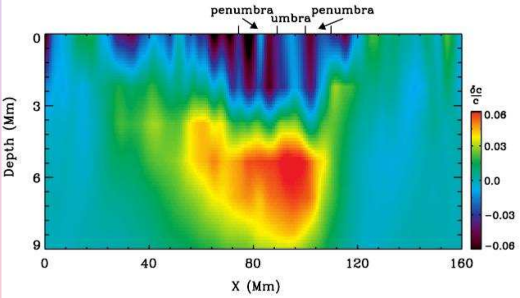
- Magnetic field needed at the bottom $\approx 10^5$ Gauss
  - $B_{\rm eq} = 10^4$ Gauss; $B_{\rm eq} = \sqrt{\mu_0 \rho}\, u_{\rm rms}$
  - buoyancy dominates
  - "tild angle" (between connecting line of 2 polarities and equator)
  - emergence latitude

Flux tubes: problems.

- Magnetic field needed at the bottom $\approx 10^5$ Gauss
- Importance of the tachocline?
  Also cycles in totally convective stars.

Flux tubes: problems.

- Magnetic field needed at the bottom $\approx 10^5$ Gauss
- Importance of the tachocline?
- Helioseismology (wave-speed anomalies)



Flux tubes: problems.

- Magnetic field needed at the bottom $\approx 10^5$ Gauss
- Importance of the tachocline?
- Helioseismology
- Avoiding distortion while raising in a turbulent media.
  - expansion of the tubes along their raise
  - thin tubes: dynamics of the surrounding medium dominates

Solve MHD equations

Let's go back to the equations!

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MHD equations

- Magnetohydrodynamics (MHD): equations for the dynamics of the plasma.
- Goal of simulations: solve MHD equations.
- Not possible to use solar parameters: $\mathrm{Re} \approx \frac{UL}{\nu} \approx 10^{10} - 10^{15}$

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Simulations

- Direct Numerical Simulations (DNS)
- Mean-field Simulations (MFS)
- Large Eddy Simulations (LES)

MFS and DNS: How to cook paella!

Mean Field Simulations (MFS)

- Quantities = averaged + fluctuations: $F = \overline{F} + f$
- Approximations: add or subtract terms in the equations.
- Control the physics.

Direct Numerical Simulations (DNS)

- Solve full equations.
- Approximations: only in resolution.
- No control.

Simulations done with Pencil Code (http://pencil-code.googlecode.com)

Full MHD equations.

Direct Numerical simulations (DNS):

- Continuity equation: $\frac{\partial \rho}{\partial t} = -\boldsymbol{\nabla} \cdot (\rho \mathbf{U})$
- Momentum equation:
$\frac{\mathrm{D}\mathbf{U}}{\mathrm{D}t} = -\boldsymbol{\nabla} p + \mathbf{J} \times \mathbf{B} + \rho \mathbf{g} + \rho \nu (\nabla^2 \mathbf{U} + \frac{1}{3} \boldsymbol{\nabla} \boldsymbol{\nabla} \cdot \mathbf{U} + 2\mathbf{S} \cdot \boldsymbol{\nabla} \ln \rho)$
- Induction equation: $\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{U} \times \mathbf{B} - \eta \mu_0 \mathbf{J})$

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Setup:

- Forcing, $k_f/k_1 = 15$
  (control scale separation)
- Strong stratification:
  density contrast ≈ 535
- $B_0/B_{\mathrm{eq}0} = 0.05$ (in range)
- 64^3x128 mesh-points
- $\mathrm{Re}_M \approx \frac{UL}{\eta} = 6$

$\Delta\overline{\mathbf{B}}/B_{\mathrm{eq}0}$ (Brandenburg et al. 2011)

Mean-Field MHD equations.

Quantities = averaged+ fluctuations: $F = \overline{F} + f$

Mean field simulations (MFS):

- Continuity equation: $\frac{\mathrm{D}\overline{\rho}}{\mathrm{D}t} = -\overline{\rho}\boldsymbol{\nabla}\cdot\overline{\mathbf{U}}$
- Momentum equation:
  $\frac{\mathrm{D}\overline{\mathbf{U}}}{\mathrm{D}t} = -c_{\mathrm{s}}^2\boldsymbol{\nabla}\ln\overline{\rho} + \mathbf{g} + \overline{\boldsymbol{\mathcal{F}}}_{\mathrm{M}} + \overline{\boldsymbol{\mathcal{F}}}_{\mathrm{K}}$
- Induction equation:
  $\frac{\partial\overline{\mathbf{B}}}{\partial t} = \boldsymbol{\nabla}\times(\overline{\mathbf{U}}\times\overline{\mathbf{B}} + \overline{\mathbf{u}\times\mathbf{b}}) + \eta\boldsymbol{\nabla}^2\overline{\mathbf{B}}$

From induction equation.

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times (\overline{\mathbf{U}} \times \overline{\mathbf{B}} + \overline{\mathbf{u} \times \mathbf{b}}) + \eta \nabla^2 \overline{\mathbf{B}}$$

α-effect (Steenbeck et al. 1966; Moffat 1978; Krause & Rädler 1980)

- Isotropic case: $\overline{\mathbf{u} \times \mathbf{b}} = \alpha \overline{\mathbf{B}} - \eta_t \overline{\mathbf{J}}$
- If $\alpha \neq 0 \rightarrow$ generate a $\overline{\mathbf{B}}$
- η_t: turbulent diffusivity.

Responsible for the Sun's large-scale field.

From momentum equation.

$$\frac{\mathrm{D}\overline{\mathbf{U}}}{\mathrm{D}t} = -c_{\mathrm{s}}^2 \boldsymbol{\nabla} \ln \overline{\rho} + \mathbf{g} + \overline{\boldsymbol{\mathcal{F}}}_{\mathrm{M}} + \overline{\boldsymbol{\mathcal{F}}}_{\mathrm{K}}$$

Mean Lorentz Force, $\overline{\boldsymbol{\mathcal{F}}}_{\mathrm{M}}$:

$$\overline{\rho}\,\overline{\boldsymbol{\mathcal{F}}}_{\mathrm{M}} = -\boldsymbol{\nabla}(\overline{\rho \mathbf{u}\mathbf{u}} + ...) = \overline{\mathbf{J}} \times \overline{\mathbf{B}} + \frac{1}{2\mu_0}\boldsymbol{\nabla}(q_{\mathrm{p}}\overline{\mathbf{B}}^2) + \ldots$$

$\overline{\mathbf{J}} = \boldsymbol{\nabla} \times \overline{\mathbf{B}}/\mu_0 = -\frac{1}{2}\boldsymbol{\nabla}\overline{\mathbf{B}}^2 + \overline{\mathbf{B}} \cdot \boldsymbol{\nabla}\overline{\mathbf{B}}$: mean current density,
$\frac{1}{2}\boldsymbol{\nabla}(q_{\mathrm{p}}\overline{\mathbf{B}}^2)$: turbulent contribution

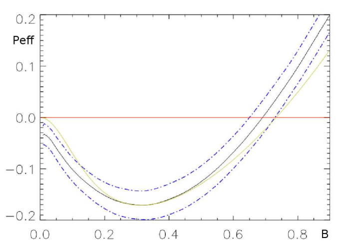
$$\overline{\rho}\,\overline{\boldsymbol{\mathcal{F}}}_{\mathrm{M}} = -\frac{1}{2}\boldsymbol{\nabla}[(1 - q_{\mathrm{p}})\overline{\mathbf{B}}^2] + \overline{\mathbf{B}} \cdot \boldsymbol{\nabla}[(1 - q_{\mathrm{s}})\overline{\mathbf{B}}]$$

Pressure. NEMPI.

Effective magnetic pressure
(Kemel et. al 2012):

(effects of turbulence on the mean Lorentz force)

$$P_{eff} = \frac{1}{2}(1 - q_p)\overline{\mathbf{B}}^2/\overline{B}_{eq}^2$$



Negative Effective Magnetic Pressure Instability (NEMPI)

- Regions below the minimum value of P_{eff} $\rightarrow$ NEMP
- NEMP + strong stratification ($|\nabla \ln \rho| > |\nabla \overline{\mathbf{B}}|$) $\rightarrow$ NEMPI [1]
- Magnetic field suppress turbulence $\rightarrow$ structures sink!

[1] Predicted: Kleeorin et al. 1989, 1990; Kleeorin & Rogachevskii 1994; Kleeorin et al. 1996; Rogachevskii & Kleeorin 2007. Confirmed: Brandenburg et al. 2011

MFS Parametrization

Normalized effective magnetic pressure:

$$\mathcal{P}_{\rm eff} = \tfrac{1}{2}(1 - q_{\rm p})\beta^2, \quad (1)$$

$q_{\rm p}(\beta)$ approximated by (Kemel et al. 2012a):

$$q_{\rm p}(\beta) = \frac{q_{\rm p0}}{1 + \beta^2/\beta_{\rm p}^2} = \frac{\beta_\star^2}{\beta_{\rm p}^2 + \beta^2}, \quad (2)$$

$q_{\rm p0}$, $\beta_{\rm p}$, and $\beta_\star = \beta_{\rm p} q_{\rm p0}^{1/2}$: constants.
$\mathcal{P}_{\rm eff}$ has a minimum value $\mathcal{P}_{\rm min}$ at $\beta_{\sf min}$, related with the parameters:

$$\beta_{\rm p} = \beta_{\sf min}^2 \Big/ \sqrt{-2\mathcal{P}_{\rm min}}, \quad \beta_\star = \beta_{\rm p} + \sqrt{-2\mathcal{P}_{\rm min}}. \quad (3)$$

Velocity field and NEMPI

Forced Velocity field

NEMPI.

$P = P_{gas} + P_{eff}$

MFS vs. DNS.

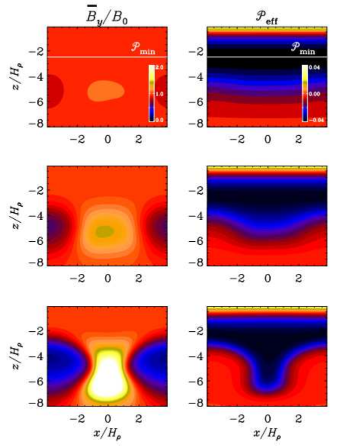


Figure 2 : Time evolution of the magnetic field and effective magnetic pressure in mean-field simulations(Kemel et al. 2012).

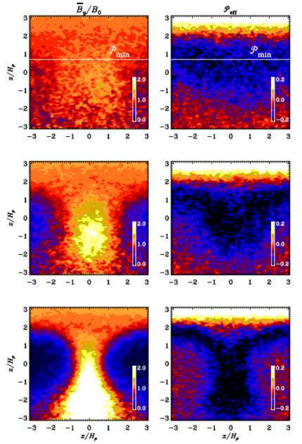


Figure 3 : Time evolution of the mean (y-averaged) magnetic field and effective magnetic pressure in direct numerical simulations (Kemel et al. 2012).

High degree of predictive power of MFS.

Different scales because of different input parameters

Previous results.

- NEMP and NEMPI described in DNS and MFS [2].
- NEMPI observed in DNS and MFS [3]
- Parameters values studied for maximize the growth strength and time:
  - $k_\mathrm{f}/k_1 = 30$ (scale separation ratio)
  - $\mathrm{Re} \equiv u_\mathrm{rms}/\nu k_\mathrm{f} = 36$
  - $P_\mathrm{m} = \nu/\eta = 0.5$.
  - $\mathrm{Re}_M = P_\mathrm{m}\mathrm{Re} = 18$
- Effects of rotation.

My contribution:

- Effects of rotation
- Effects of stratification
- Effects of corona: 2 layers model

[2] Kleeorin et al. 1989, 1989, 1996; Kleeorin & Rogachevskii 1994;

Rotation. Structures evolution

(Rotational effects on the negative magnetic pressure instability (2012, A&A,548,A49))

(Competition of rotation and stratification in flux concentrations (2013, A&A, 556, A83))

$$\mathrm{Co} = 2\Omega\tau_c = 2\Omega/u_{\mathrm{rms}}k_{\mathrm{f}}$$

$\mathrm{Co} = 0.006$ $\mathrm{Co} = 0.03$ $\mathrm{Co} = 0.06$

Simulations done with Pencil Code (http://pencil-code.googlecode.com)

MFS and DNS results.

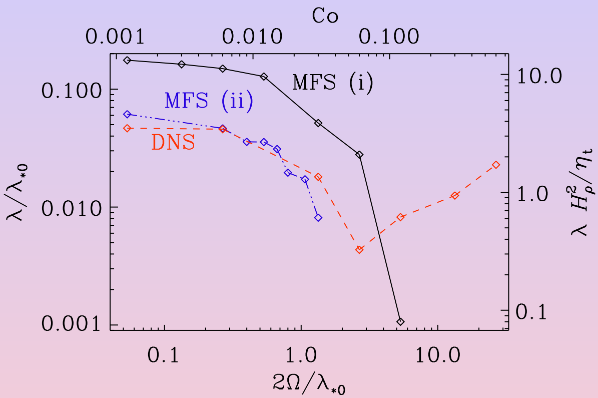


Figure 4 : MFS (i): $q_{\rm p0} = 20$ and $\beta_{\rm p} = 0.167$; MFS (ii): $q_{\rm p0} = 32$ and $\beta_{\rm p} = 0.058$.

$B_0/B_{\rm eq0} = 0.05$. $\quad q_p = \frac{\beta_\star^2}{(\beta_{\rm p}^2+\beta^2)}$ $\quad \beta = \frac{\overline{\mathbf{B}}}{B_{\rm eq0}}$ $\quad \lambda_{*0} \equiv \beta_\star u_{\rm rms}/H_\rho$

Polytropic stratification. Horizontal imposed field (MFS).

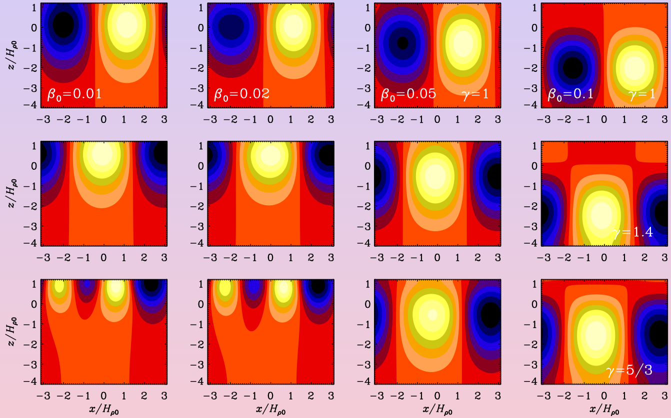


Figure 5 : $\overline{B}_y$ in the kinematic growth phase for different γ and β_0 in the presence of a horizontal field using the perfect conductor boundary condition.

Polytropic stratification. Vertical imposed field (MFS). Evolution.

$\gamma = 1$ $\gamma = 5/3$

$\beta_0 = 0.02$

Vertical field. Sunspot-like structure

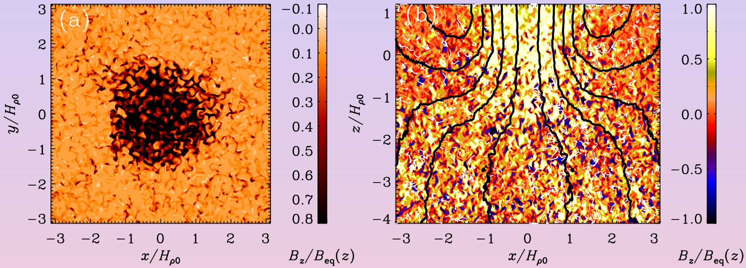


Figure 6 : Cuts of $B_z/B_{\rm eq}(z)$ in the xy plane at the top boundary ($z/H_{\rho 0} = 1.2$) and the xz plane through the middle of the spot at $y = 0$ for $\gamma = 5/3$ and $\beta_0 = 0.05$. In the xz cut, we also show magnetic field lines and flow vectors obtained by numerically averaging in azimuth around the spot axis.

Vertical imposed field (DNS vs MFS). Evolution.

DNS

MFS

$\beta_0 = 0.05$

Parameters dependence study in polytropic stratification

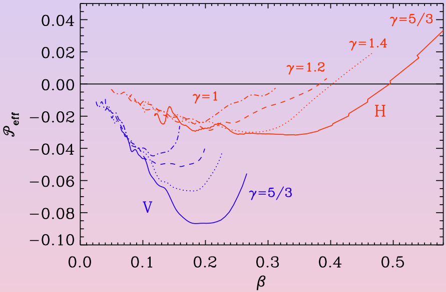


Figure 7 : Effective magnetic pressure obtained from DNS in a polytropic layer with different γ for horizontal (H, red curves) and vertical (V, blue curves) mean magnetic fields.

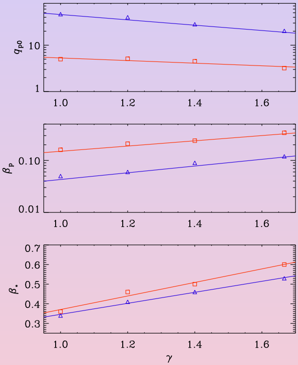


Figure 8 : Parameters q_{p0}, β_{p}, and $\beta_\star$ for the function $q_{\text{p}}(\beta)$ versus γ for horizontal (red line) and vertical (blue line) mean magnetic fields.

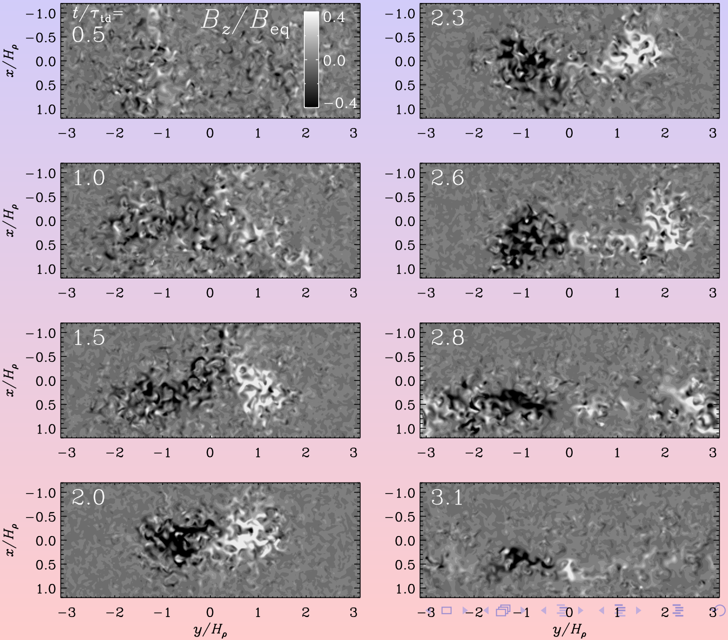
Corona: two layers model

Bipolar magnetic structures driven by stratified turbulence with a coronal envelope (2013, ApJ Lett 777, L37; arXiv:1308.1080)

Bipolar magnetic structures.

$256^2 \times 512$, $kf/k1 = 30$, $\mathrm{Re}_M = 18$

Bipolar magnetic structures



Conclusions

Parker instability vs. NEMPI

	NEMPI	**Parker Instability**
Turbulence	turbulent	non turbulent
Scale	sufficient many eddies	small
Stratification	continuous strat. B	non uniform & initially separated
Energy source	turbulent energy	gravitational field
Initial field	smooth: $H_B^{-1} \equiv \|\nabla \ln B\|$ small	structured: $\|\nabla \ln B\|$ large
Density variation	$H_\rho^{-1} \equiv \|\nabla \ln \rho\|$ large	$H_\rho^{-1} \equiv \|\nabla \ln \rho\|$ small
Instability criteria	$\frac{H_B}{H_\rho} \gg 1$	$\frac{H_B}{H_\rho} \ll 1$

Conclusions.

Rotation.

NEMPI depends on the ratio between rotation and turbulence.

- Turnover time: $\tau \approx 2$hours
- On the Sun: only upper-most layers (super-granulation layer $\tau \sim 1 day$)

NEMPI + Dynamo:

- Coupled system for $\mathrm{Co} > 0.1$
- Direct comparison with NEMPI growth rate not possible.

Conclusions.

Polytropic EoS:

- NEMPI develops in the uppermost layers.
- The parametrization depends on the polytropic index.
- Isothermal models applicable locally.
- No “potato-sack” effect with vertical fields.

Coronal envelope:

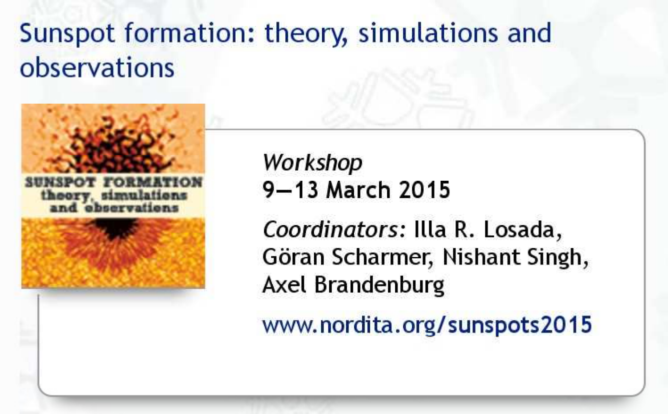
- Emergence and decaying of bipolar magnetic region.
- The 2 polarities tend to separate.

Solar activity as a surface phenomenon.

What is next?.

- Effects of rotation in a vertical field configuration.
- NEMPI dependence on *Re*.
- MFS studies of the two-layer model.
- Polytropic atmosphere in the two-layer model.
- ....

Sunspot formation: theory, simulations and observations



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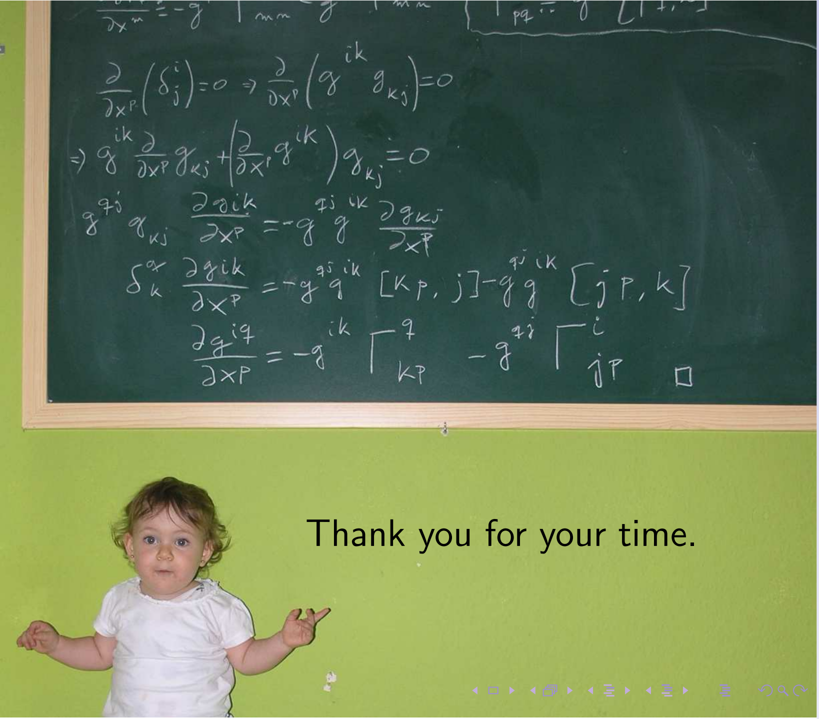

Thank you for your time.